

Polarimetric Spatio-Temporal Light Transport Probing

- Supplemental Document -

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ACM Reference Format:

Seung-Hwan Baek and Felix Heide. 2021. Polarimetric Spatio-Temporal Light Transport Probing - Supplemental Document - . *ACM Trans. Graph.* 0, 0, Article 0 (2021), 17 pages. <https://doi.org/http://dx.doi.org/>

In this Supplemental Document, we present additional details on the method and analysis presented in the main manuscript, and we provide additional assessment validating the approach. Specifically, the document covers

- Comparison with existing light transport imaging methods.
- Spatio-polarimetric shape reconstruction.
- Mueller matrices of optical elements.
- Spatio-temporal polarimetric path tracer.
- Low-dimensional angular-polarimetric embeddings.
- Learned rotating ellipsometry.
- Polarimetric reconstruction.
- Polarimetric decomposition.
- Geometric calibration.
- Polarimetric calibration.
- Hardware part list for the prototype systems.
- Additional results.

We also refer the reader to the Supplemental Video for further reconstruction results and illustrations of the capture process.

1 COMPARISON WITH EXISTING LIGHT TRANSPORT ACQUISITION METHODS

Table 1 provides an overview of the transport dimensionality resolved by existing light transport acquisition methods compared to the proposed one.

2 SPATIO-POLARIMETRIC SHAPE RECONSTRUCTION

We propose a shape-reconstruction method for objects with strong subsurface scattering. To this end, we capture approximate direct reflections using epipolar imaging combined with structured-light scanning and shape from polarization.

Epipolar Structured Light. We capture an epipolar image for each of the graycode patterns from which depth is reconstructed with a conventional graycode decoding method. Specifically, we use OpenCV's `getProjPixel()` function of the graycode class for the decoding. The depth map is then unprojected to global coordinate followed by filtering out outliers and surface normal estimation

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with principle component analysis. We use MATLAB's `pcdenoise()` for denoising and `pcnormals()` for point-cloud normal estimation. We project the resulting 3D points with the estimated normals back to the camera screen space and inpaint pixels without normals using a Laplacian-based method [Levin et al. 2007]. Note that the missing pixels originate from the denoising process on the 3D points. This results in the base surface normals $\mathbf{N}_b$ computed from the epipolar graycode patterns.

Epipolar Shape from Polarization. Conventional shape from polarization (SfP) methods often suffer from interreflections and subsurface scattering. In our work, we combine SfP with epipolar spatial scanning so that surface normals can be estimated only from the approximate direct reflections. SfP relies on Fresnel theory to estimate zenith angle θ of the surface normal with the refractive index η and the degree of polarization ρ as

$$\theta = f(\eta, \rho). \quad (1)$$

Refer to Kadambi et al. [2015] for the definition of $f()$. We use $\eta = 1.6$ in our experiment. We compute the degree of polarization of the spatio-polarimetric light transport $\int_t \mathcal{T}(s, s_e, :, t) \mapsto \mathbf{M}$ as

$$\rho = \frac{\sqrt{\mathbf{M}(1, 0)^2 + \mathbf{M}(0, 1)^2}}{\mathbf{M}(0, 0)}. \quad (2)$$

We also compute the azimuthal angle ϕ of the surface normal with π ambiguity as

$$2\phi = -\tan^{-1} \frac{\mathbf{M}(2, 0)}{\mathbf{M}(1, 0)}. \quad (3)$$

Combining Spatial and Polarimetric Cues. We combine the surface normals from the epipolar structured light and SfP. To this end, we employ the optimization-based solution proposed in Baek et al. [2018] that combines regular structured light and SfP as

$$\begin{aligned} \underset{\mathbf{N}}{\text{minimize}} \quad & w_p \|\mathbf{W}(\mathbf{V}\mathbf{N} - \mathbf{Z})\|_2^2 + w_p \|\mathbf{W}\mathbf{A}\mathbf{N}\|_2^2 \\ & + w_b \|\mathbf{G}\mathbf{N} - \mathbf{N}_b\|_2^2 + w_s \|\mathbf{V}\mathbf{N}\|_2^2, \end{aligned} \quad (4)$$

where $\mathbf{W}$ is a boolean confidence matrix set to one if the DoP is smaller than 0.1, otherwise zero. $\mathbf{V}$ is the viewing vector matrix assuming the orthogonal projection. $\mathbf{Z}$ is the cosine zenith matrix proposed in Baek et al. [2018] as $\cos(\theta)$. $\mathbf{A}$ is the collinearity azimuth matrix as $[\cos \phi, -\sin \phi, 0]$ proposed in Tozza et al. [2017]. $\mathbf{G}$ is the Gaussian blur matrix with the kernel size of 50 pixels. w_p, w_b, w_s are the balancing terms as 10, 10, and 1e-11. As Equation (4) is a convex problem, we solve it efficiently with conjugate gradient optimization. The estimated surface normals is then normalized with the unit-vector constraint and we solve Equation (4) again with the estimated normals as initial points. This procedure is repeated for 30 iterations.

Probing	Space	Time	Polarization	Total Dimension
[Treibitz and Schechner 2008]	2D	equilibrium	cross polarization	2
[Azzam 1978]	1D	equilibrium	complete polarization	2
[Velten et al. 2013]	2D	femtoseconds	intensity only	3
[O'Toole et al. 2018]	2D	picoseconds	intensity only	3
[Kotwal et al. 2020]	4D	femtoseconds	cross polarization	4
[Baek et al. 2018]	2D	equilibrium	linear polarization	4
[O'Toole et al. 2012]	4D	equilibrium	intensity only	4
[O'Toole et al. 2014]	4D	nanoseconds	intensity only	5
Proposed Temporal-Polarimetric Probing	2D	picoseconds	complete polarization	5
Proposed Spatio-Polarimetric Probing	4D	equilibrium	complete polarization	6

Table 1. We compare our proposed light transport probing method to previous approaches. Our temporal-polarimetric and spatio-polarimetric setup enable complete polarimetric acquisition of light transport along with temporal and spatial dimensions respectively.

3 MUELLER MATRICES OF OPTICAL ELEMENTS

We provide the Mueller matrices of optical elements used in the main paper.

3.1 Linear Polarizer

Linear polarizer at angle θ is modelled as

$$\mathbf{L}(\theta) = \frac{1}{2} \begin{bmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

3.2 Quarter Wave Plate

Quarter wave plate at angle θ has the Mueller matrix form

$$\mathbf{Q}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta & \sin 2\theta \cos 2\theta & -\sin 2\theta \\ 0 & \sin 2\theta \cos 2\theta & \sin^2 2\theta & \cos 2\theta \\ 0 & \sin 2\theta & -\cos 2\theta & 0 \end{bmatrix}. \quad (6)$$

Note that we consider the calibrated retardance of the real QWP (Thorlabs AQWP10M-580) instead of using the quart-wave assumption. For the temporal-polarimetric setup, we use the wavelength of 635nm and the corresponding calibrated retardance of the QWP is 0.24. For the spatio-polarimetric setup, the central trichromatic wavelengths (470, 525, 645 nm) are mapped to the calibrated retardance values of 0.25, 0.24, 0.24, respectively.

3.3 Non-polarizing Beam Splitter

We employ a non-polarizing beamsplitter (NPB) of Thorlabs BS013 that has minimal impact on the polarization of the light. The ideal Mueller matrices of the NPB in reflection and transmission modes are as follows, see also Liu et al. [2016] for its potential deviation

$$\mathbf{B}_T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B}_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (7)$$

3.4 Galvo Scanning

For spatial scanning of a scene, we use a 2-axis galvo mirror system (Thorlabs GVS012) which is coated with protected silver with high reflectivity at both parallel and perpendicular components of the electric field. While 2D galvo scanning is essential for spatial scanning of a scene, it could affect the polarization state of light at different angles of the galvo mirrors. However, it is challenging to experimentally calibrate this change due to the large set of possible angles and the hardware configuration limits us to orient the analyzing modules corresponding to all the tilted angles. Therefore, we analytically model the polarization change induced by galvo mirrors based on Fresnel reflection following Petrova-Mayor and Knudsen [2017]. Our mirror is made of silver which has a complex-valued refractive index $n+ki$, where n, k are the real-valued refractive index and the extinction coefficient as the imaginary component. When light wave is incident on the silver mirror at the incident angle of θ_i , the polarization state of the reflected light changes by the Fresnel Mueller matrix [Baek et al. 2018]. Our galvo mirror operates by supplying a pair of voltage values to the controller corresponding to a specific angle of the output beam. The maximum resolution is 0.0008 degrees and the range is ± 20 degrees according to the specification.

4 SPATIO-TEMPORAL POLARIMETRIC PATH TRACER

To aid the analysis and intuition of light transport tensor, we have implemented a path tracer that allows us to simulate the 7D light transport $\mathcal{T}$ using the Mitsuba2 renderer [Nimier-David et al. 2019]. To simulate spatial imaging, we place a 2D emitter and a 2D detector and pixels are sampled in their screen spaces. That is, we sample $uv \in [0, 1]^2 \sim \mathbb{S}_{\text{sensor}}$, $st \in [0, 1]^2 \sim \mathbb{S}_{\text{emitter}}$ as emitter and detector coordinates, and we only consider those that satisfy $\delta - \epsilon < st - uv < \delta$, where $\delta \in [0, 1]^2$, $\epsilon \in [0, 2]^2$. This means that we can specify a certain range of pixels on the light source to illuminate the scene, and we can specify specific pixel on the camera to sample the transport. Temporal rendering is based on the time-of-flight from the emitter to the sensor. We refer to [Jarabo et al. 2014; Pediredla et al. 2019] for recent advances and overview in transient rendering. Only photons that satisfy $t_{\min} < t_{\text{time of flight}} < t_{\max}$ are rendered

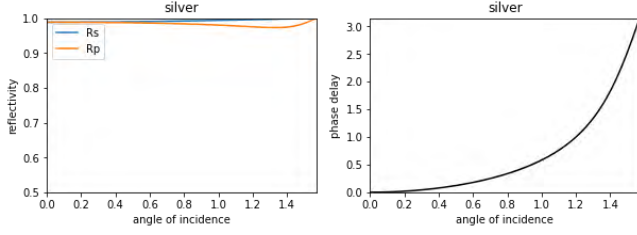


Fig. 1. We use a silver-coated galvo mirror suitable for polarization analysis which has minimal impact on the polarization state of reflected light. The galvo mirror has high reflectivity for both vertical and parallel polarization states and the phase delay induced by the galvo reflection is insignificant for galvo working range from -20 to 20 degrees. Nevertheless, we consider this polarimetric change using a Fresnel-reflection model [Petrova-Mayor and Knudsen 2017].

This gating constraint can be used to only illuminate surfaces at certain summed distance between the camera and the emitter. For polarization rendering, we use the vector path integral [Jarabo and Arellano 2018]. The polarization state is controlled through a virtual polarizing dual-rotating retarder setup.

5 LOW-DIMENSIONAL ANGULAR-POLARIMETRIC TRANSPORT EMBEDDINGS

In the main paper, we analyzed the low-dimensional embeddings of polarimetric light transport. Here, we report another analysis on angular-polarimetric light transport which combines the polarimetric analysis presented in the main manuscript and the angular-analysis method of Nielsen et al. [2015].

First we arrange the normalized and compressed polarimetric BRDFs as proposed in the main paper. This forms an observation matrix $\mathbf{X} \in \mathbb{R}^{25 \times N}$, where N is the number of valid bins in each polarimetric BRDF. Principal components of $\mathbf{X}$ are then extracted using singular value decomposition

$$\mathbf{X} - \mu = \mathbf{U}\Sigma\mathbf{V}^T, \quad (8)$$

where μ is the mean matrix of $\mathbf{X}$ over different materials. $\mathbf{U}$ and $\mathbf{V}$ contain the eigenvectors and Σ is a matrix with the eigenvalues on its diagonal entries. Figure 2 shows the first three principal components with two different angular slices of (θ_d, θ_h) and (ϕ_d, θ_h) based on Rusinkiewicz parameterization [Rusinkiewicz 1998]. We use the pBRDF dataset of Baek et al. [2020] while Nielsen et al. [2015] used the intensity BRDF dataset [Matusik 2003].

As a sanity check, we confirm that our analysis results in similar principal components of intensity BRDF as Nielsen et al. [2015]. The three principal components of intensity BRDF exhibit specular reflections, diffuse reflections, and specular shapes respectively. This low-dimensional embedding of intensity BRDF has fueled many applications such as efficient BRDF acquisition [Nielsen et al. 2015] and BRDF editing [Sun et al. 2018].

Next, we examine the principal components of angular-polarimetric dimensions. We found that low-frequency angular-polarimetric variation is observable for diffuse reflection in all of the principal components. This indicates a close link between the diffuse polarization

and the surface geometry. Furthermore, Fresnel peaks present notable angular-polarimetric variations around the Brewster angles which might be related to the material's refractive index. Lastly, angular-polarimetric principal components have variation along the ϕ_d axis different from the angular analysis of intensity BRDFs.

6 LEARNED ROTATING ELLIPSOMETRY

The proposed ellipsometry method learns the optimal angles of the polarizing optics to maximize reconstruction accuracy on real-world polarimetric light transport. This concept can also be applied to the existing dual rotating retarder (DRR) method without any change on the hardware setup, by optimizing the angle configurations for a given number of total captures. Figure 3 shows an illustration of our learned dual-rotating retarder (DRR) and learned quad-rotating polarizer retarder (L-QRPR) setup. The optimized angles for the DRR setup are different from the original configuration of the DRR with substantial variation of the angles. Our learned method for the quad-rotating polarization retarder setup shows notable difference with periodic variations of the optimized angles. The same trends also applies to each configuration's extension with a polarizer-array sensor.

To evaluate the impact of Poisson noise, we also simulated training samples with Poisson-Gaussian noise instead of Gaussian-only noise in Equation (12) of the main manuscript. We found that incorporating Poisson noise caused the learned angles deviate by an additional ~ 0.5 radians on average. This indicates that noise statistics affects the learning procedure. Investigating noise-sensitive learned ellipsometry for photon-deficient environments is an interesting avenue for future research.

7 POLARIMETRIC RECONSTRUCTION

We stack the measured intensity I for different retarder angles in a vector as $\mathbf{I}$. We then formulate an optimization problem in the form of a normal equation: $\mathbf{S}\mathbf{M}_{\text{vec}} = \mathbf{I}_c$, where $\mathbf{S}$ is a system matrix and $\mathbf{M}_{\text{vec}}$ the vectorized Mueller matrix to be estimated. The system matrix $\mathbf{S}$ is constructed by $\mathbf{S} = \hat{\mathbf{A}} \otimes \hat{\mathbf{P}}$, where $\hat{\mathbf{A}}$ is the analyzing matrix and $\hat{\mathbf{P}}$ is the polarizing matrix which only includes the Mueller matrices of the optical elements without multiplying the Stokes vector of the laser illumination. The analyzing and the polarizing matrices are differently defined for each setup based on their image formation models. For the spatio-polarimetric setup, $\hat{\mathbf{A}}$ is the stack of the first row of $\mathbf{A}^{\theta_k^1, \theta_k^3}$ for all measurements $k \in 1, \dots, K$: $\mathbf{A}^{\theta_k^1, \theta_k^3} = \mathbf{L}^{\theta_k^1} \mathbf{Q}^{\theta_k^3}$. Note that, when we use the polarizer-array sensor in the spatio-polarimetric setup, the analyzing matrix has the four-times more observations, which means its number of rows is $4K$ instead of K . The polarizing matrix $\hat{\mathbf{P}}$ is the stack of the first column of $\mathbf{P}^{\theta_k^2, \theta_k^1} = \mathbf{Q}^{\theta_k^2} \mathbf{L}^{\theta_k^1}$. For the temporal-polarimetric setup, their definitions of the analyzing and the polarizing matrices change according to the image formation models: $\mathbf{A}^{\theta_k^1, \theta_k^3} = \mathbf{L}^{\theta_k^1} \mathbf{Q}^{\theta_k^3} \mathbf{B}_r \mathbf{G}^s$, and $\mathbf{P}^{\theta_k^2, \theta_k^1} = \mathbf{G}^s \mathbf{B}_t \mathbf{Q}^{\theta_k^2} \mathbf{L}^{\theta_k^1}$.

8 POLARIMETRIC DECOMPOSITION

Our spatio-polarimetric acquisition method allows for an analysis of light transport through polarimetric decomposition. Figure 4

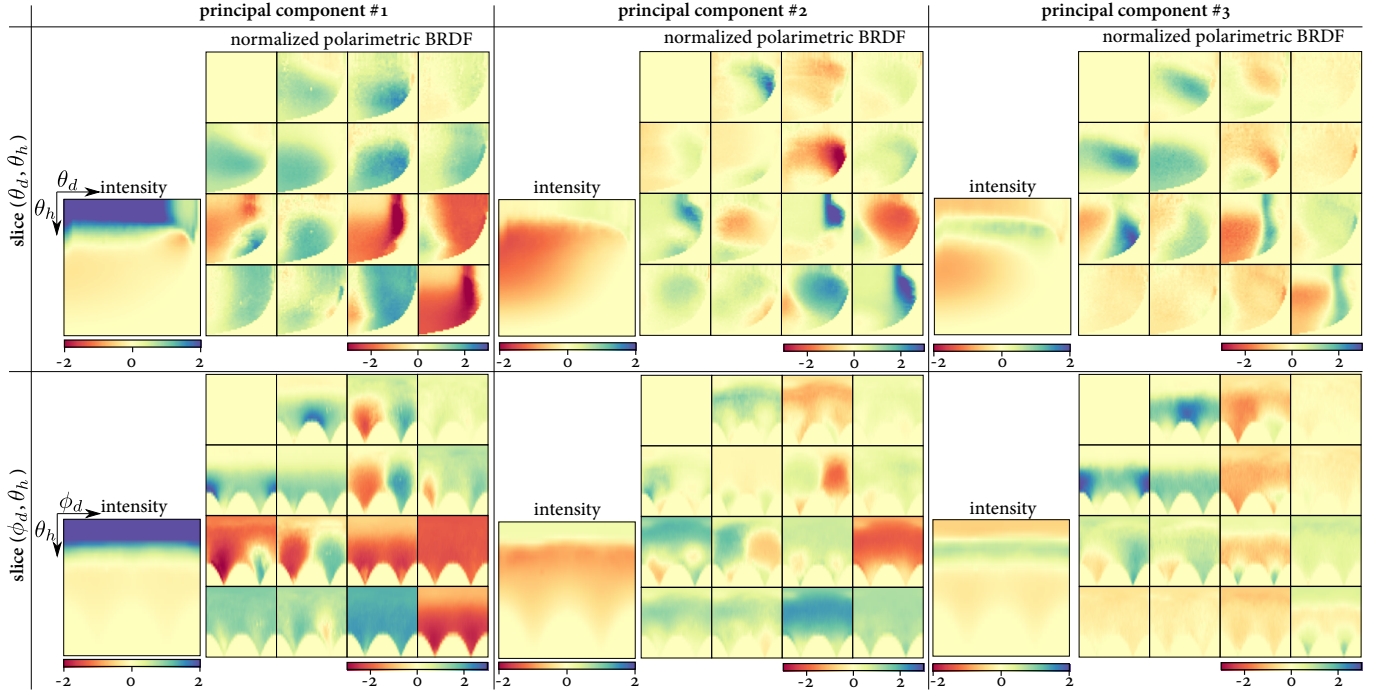


Fig. 2. We estimate and visualize the top-three principal components of the normalized angular-polarimetric light transport as well as the intensity dimension. We found low-dimensional embeddings of angular-polarimetric BRDF which exhibit unique characteristics different from the patterns observed in the angular analysis of intensity BRDF. See text for details.

visualizes the estimated diattenuation, polarizance, and retardance at the green spectral channel for epipolar and non-epipolar imaging. This auxiliary information encodes distinctive crystal properties, notably the birefringence of calcite and quartz in the retardance estimates.

9 GEOMETRIC CALIBRATION

Spatio-polarimetric Setup. In order to probe the directional light transport between the projector and the camera, we calibrate the geometric parameters. We first capture a checkerboard with different poses to obtain accurate intrinsic parameters of the camera. We then project structured illumination using the projector for different checkerboard poses and estimate the intrinsic parameters of the projector. Given the estimated intrinsics, we undistort both the camera and the projector views from which the fundamental matrix between these undistorted views are estimated. We obtain the reprojection errors of 0.5 px, 0.2 px, 0.7 px for the camera intrinsics, the projector intrinsics, and the stereo extrinsics, respectively.

Coaxial Temporal-polarimetric Setup. We establish the mapping from the world coordinates of light coming out of the galvo mirrors and the voltage inputs to the galvo system, see Figure 5. We use a Thorlabs TPSM2/M galvo, a laser-safe screen with corner points at known grid positions. We manually adjust the voltage values applied to the galvo mirrors in order to direct the beam to a specific corner point on the grid dots. The distance between the screen to the exit surface of the galvo cage is 5.5 cm. Once the data is obtained

for multiple grid points, we fit a third-order polynomial to map the world coordinates to the voltage values. The R-square value was 0.9999. We specify grid points on the scene to be captured which are then converted to the corresponding voltages with the calibrated mapping.

Quad-rotating Ellipsometry and Polarizer-array. We implement the proposed quad-rotating ellipsometry and its extension with a polarizer-array camera. The polarizer-array camera is calibrated for its intrinsic and extrinsic parameters with respect to the projector. We use the demosaiced image using bilinear interpolation as the input to our method. The reprojection errors are 0.6 px, 0.4 px, 0.8 px for the polarization camera intrinsics, the projector intrinsics, and the stereo extrinsics.

10 POLARIMETRIC CALIBRATION

Spatio-polarimetric Setup. We align the fast axes of the two quarter-wave plates and the polarizing axes of the two linear polarizers. To this end, we perform extinction-based calibration by orienting each optical element sequentially. First, a reference linear polarizer with the horizontal polarizing axis is installed in front of the projector. Another linear polarizer is placed while ensuring the minimum intensity of the transmitted light by adjusting its angle, resulting in the vertical polarizing axis. Next, we place a QWP in between the linear polarizers. The angle of the QWP is adjusted to minimize the intensity of the light. The last QWP is then inserted instead of the calibrated QWP and follow the same procedure. Note that

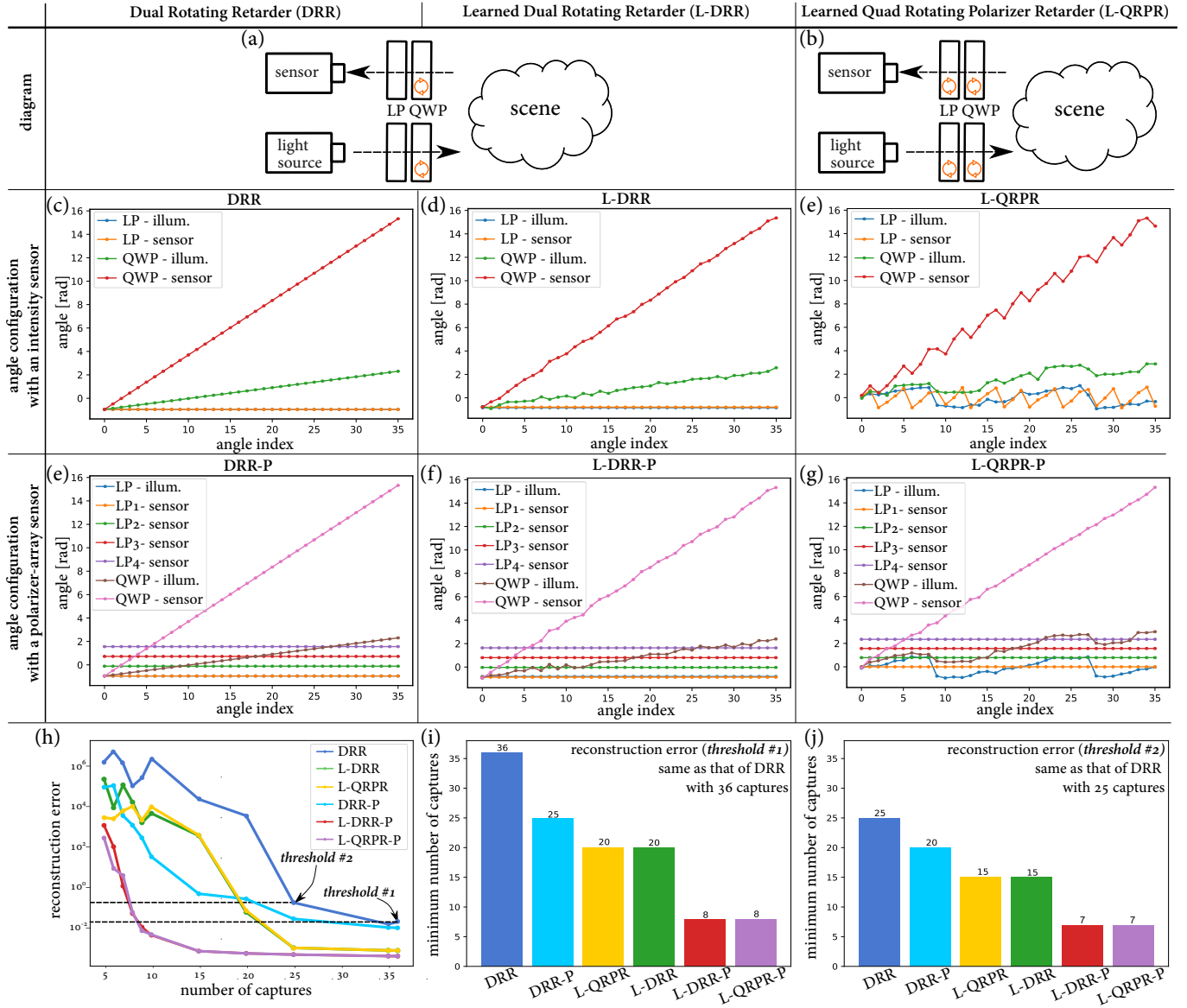


Fig. 3. We learn the optimal angles of the rotating polarizing optics for the conventional dual rotating retarder setup and the proposed quad-rotating polarizer retarder setup. In addition, we also extend these methods to an emerging polarizer-array sensor. We perform 5-fold cross validation and found that the number of required rotations in the conventional setup can be reduced by more than 2× when using the learned angles and by more than 3× when using a polarizer-array sensor.

the QWPs have calibrated indicators which remove the ambiguity of the slow-fast axes. As the two QWPs fast axes are aligned to the vertical, we finally rotate the reference linear polarizer by 90 degrees.

Coaxial Temporal-polarimetric Setup. We align the fast axes and polarization axes of the QWPs and LPs as in spatio-polarimetric setup first without using the beamsplitter and the galvo mirror. Note that we intentionally misaligned optical elements with marginal angles to minimize interreflections which gives us the correct Mueller

matrix estimates of air as the identity Mueller matrix. Then, we place the non-polarizing beamsplitter in transmission mode and verify that we obtain the identity Mueller matrix. We also test the reflection configuration of the beamsplitter and validated that the Mueller matrix is a diagonal matrix with the [1,1,-1,-1] diagonal vector. Next step is to place a mirror in the scene, considering the Mueller matrix of the beamsplitter, we obtain the identity Mueller matrix.

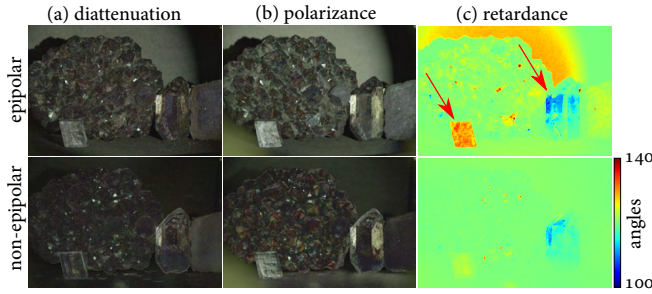


Fig. 4. We analyze polarimetric properties of crystals with epipolar and non-epipolar imaging including (a) diattenuation, (b) polarizance, and (c) retardance. This reveals the birefringence properties of crystals indicated by red arrows in the retardance. The two crystals are Calcite (left) and Quartz (right).

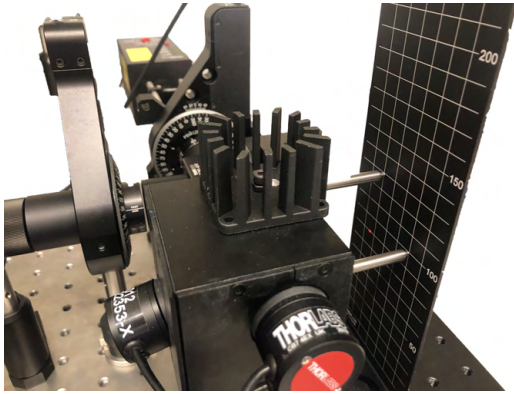


Fig. 5. We calibrate the mapping between the voltage applied to the galvo mirror and its directed beam angle based on known grid locations at a fixed distance from the galvo exit.

Quad-rotating Ellipsometry and Polarizer-array. We first perform the extinction-based polarimetric calibration procedure from above. One major difference is that we need to obtain the orientation of the polarizer-array filters inside of the polarizer-array camera. To this end, we find an optimal angle for each polarizer-array filter by rotating an additional linear polarizer in front of the camera and find the extinction of the captured signals.

11 QUAD-ROTATING ELLIPSOMETRY FOR TEMPORAL-POLARIMETRIC IMAGING

We demonstrate our learned quad-rotating ellipsometry in the spatio-polarimetric setup. Applying this same learned configuration to the temporal-polarimetric setup would require employing an additional half-wave plate in front of the laser module. As the laser illumination itself is linearly polarized, we can implement the same effect of rotating a linear polarizer by rotating the half-wave plate in front of the linearly-polarized laser module. The angles of the half-wave plate can be jointly learned via our optimization procedure by only replacing the Mueller matrix of the laser-side linear polarizer with

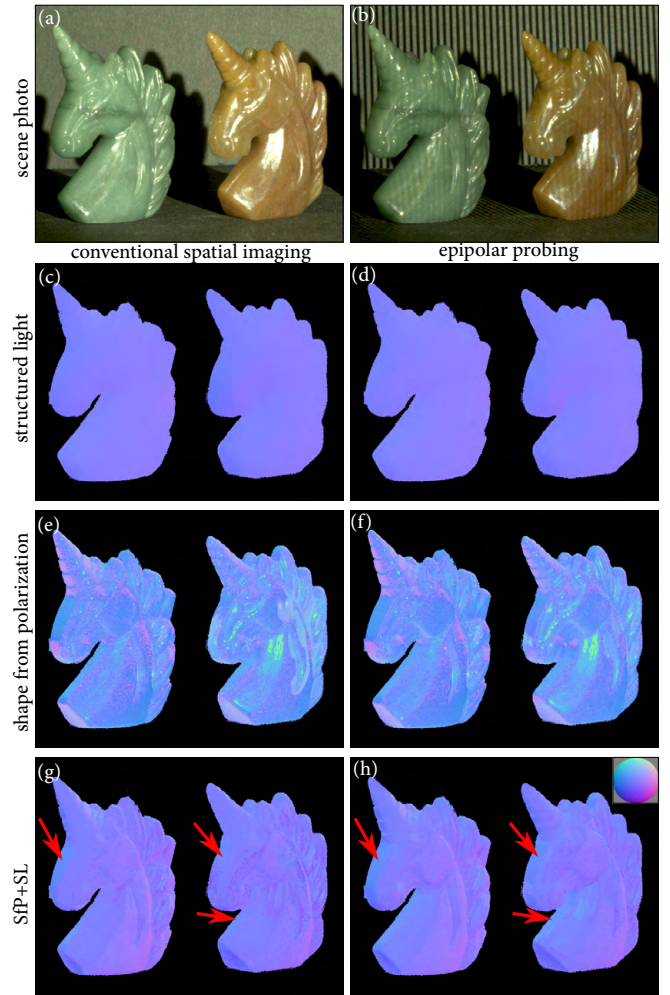


Fig. 6. We capture (a) two unicorn statues that have strong scattering appearance as shown in (b) the gray-code projection. (c) & (d) Conventional structured light (SL) and epipolar probing fail with this strong subsurface scattering, resulting in incorrectly smoothed surface normals. (e) & (f) Extracting surface normals from polarization, so-called shape from polarization (SfP), reveals geometric details, however at the cost of substantial normal ambiguity. (g) Directly combining structured-light scan with the polarimetric imaging fail to identify correct surface normals for challenging regions as indicated by the red arrows [Baek et al. 2018]. (h) Our method jointly exploits epipolar spatial and polarimetric probing, resulting in accurate geometric reconstruction.

that of the half-wave plate. We leave this engineering effort as interesting future work.

12 HARDWARE PART LIST FOR THE PROTOTYPE SYSTEMS

Table 2 lists all of the parts used in our prototype systems.

Setup	Item #	Part description	Quantity	Model name
Spatio-polarimetric (DRR)	1	Machine vision camera	1	FLIR GS3-U3-51S5C
	2	Objective lens	1	Fujinon HF25HA-1S 25mm f/1.4
	3	DMD projector	1	Lightcraft LC3000-G2-Pro
	4	Linear polarizer	2	Newport 10LP-VIS-B
	5	Achromatic quarter-wave retarder	2	Thorlabs AQWP10M-580
	6	Manual rotary stage	2	Newport RM25A
	7	Motorized rotary stage	2	Thorlabs KPRM1E
	8	Adjustable angle plate	1	Thorlabs AP180
Spatio-polarimetric (L-QRPR)	1	Machine vision camera	1	FLIR GS3-U3-51S5C
	2	Objective lens	1	Fujinon HF25HA-1S 25mm f/1.4
	3	DMD projector	1	Lightcraft LC3000-G2-Pro
	4	Linear polarizer	2	Newport 10LP-VIS-B
	5	Achromatic quarter-wave retarder	2	Thorlabs AQWP10M-580
	6	Motorized rotary stage	4	Thorlabs KPRM1E
	7	Adjustable angle plate	1	Thorlabs AP180
Spatio-polarimetric (L-QRPR-P)	1	Polarizer-array camera	1	FLIR Blackfly S BFS-U3-51S5PC
	2	Objective lens	1	Fujinon HF25HA-1S 25mm f/1.4
	3	DMD projector	1	Lightcraft LC3000-G2-Pro
	4	Linear polarizer	2	Newport 10LP-VIS-B
	5	Achromatic quarter-wave retarder	2	Thorlabs AQWP10M-580
	6	Motorized rotary stage	4	Thorlabs KPRM1E
	7	Adjustable angle plate	1	Thorlabs AP180
Coaxial temporal-polarimetric (DRR)	1	Singe-photon avalanche diode	1	MPD
	2	Time-correlated single photon counting	1	PicoQuant TimeHarp 260 PICO
	3	Objective lens	1	Canon EF 50mm f/1.8 STM
	4	Linear polarizer	1	Newport 10LP-VIS-B
	5	Achromatic quarter-wave retarder	2	Thorlabs AQWP10M-580
	6	Manual rotary stage	1	Newport RM25A
	7	Motorized rotary stage	2	Thorlabs KPRM1E
	8	Non-polarizing beamsplitter	1	Thorlabs BS013
	9	Beamsplitter cage	1	Thorlabs CCM1-4ER
	10	Dielectric mirror	1	Thorlabs BB1-E02
	11	Right-angle mirror mount	1	Thorlabs KCB1
	12	Beam block	1	Thorlabs LB1
	13	Cage aperture	1	Thorlabs CP20S
	14	Picosecond pulsed diode laser	1	Edinburgh Instruments EPL-635
	15	Galvo mirror	1	Thorlabs GVS012
	16	Galvo power supply	1	GPS011-US
	17	EOS-to-C-mount adaptor	1	Fotodiox pro EOS (α)-C-D-Click

Table 2. List of components used for our acquisition systems.

13 ADDITIONAL RESULTS

Multi-Bounce Decomposition. Figure 8 shows an additional result validating that the proposed method is capable of decomposing complex reflections between two coins and diffuse planes.

Learned Ellipsometry. Figure 9 shows that our learned ellipsometry outperforms conventional DRR, achieving same reconstruction accuracy with fewer captures. Our learned ellipsometry, specifically L-QRPR-P, provides consistent results with 15, 20, and 36 captures, while conventional DRR results in reconstruction artifacts for 20 and 15 captures. We note that color inconsistency between methods originates from the different spectral responses of the intensity

sensor and the polarizer-array sensor. Last, Figure 10 shows the reconstructed spatio-polarimetric transport of both epipolar and non-epipolar components with the proposed L-QRPR-P method using 15 captures.

Shape Reconstruction with Subsurface Scattering. We exploit spatial and polarimetric information in the light transport tensor in order to reconstruct the shape of highly-scattering objects. In addition to the dragon object presented in the main paper, we show an additional result on the two unicorn heads with scattering properties as shown in Figure 6. Our method outperforms existing structured light scanning, shape from polarization, and their combination [Baek

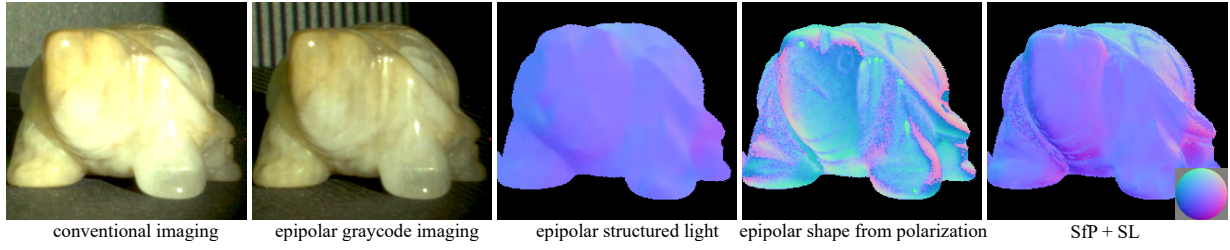


Fig. 7. Our spatio-polarimetric imaging allows for shape reconstruction of a turtle sculpture with strong subsurface-scattering appearance. Existing approaches such as epipolar structured light and epipolar shape-from-polarization both provide inaccurate and incomplete geometric reconstructions. The proposed method exploits the benefits of both approaches and can therefore successfully reconstruct coarse to fine geometric details.

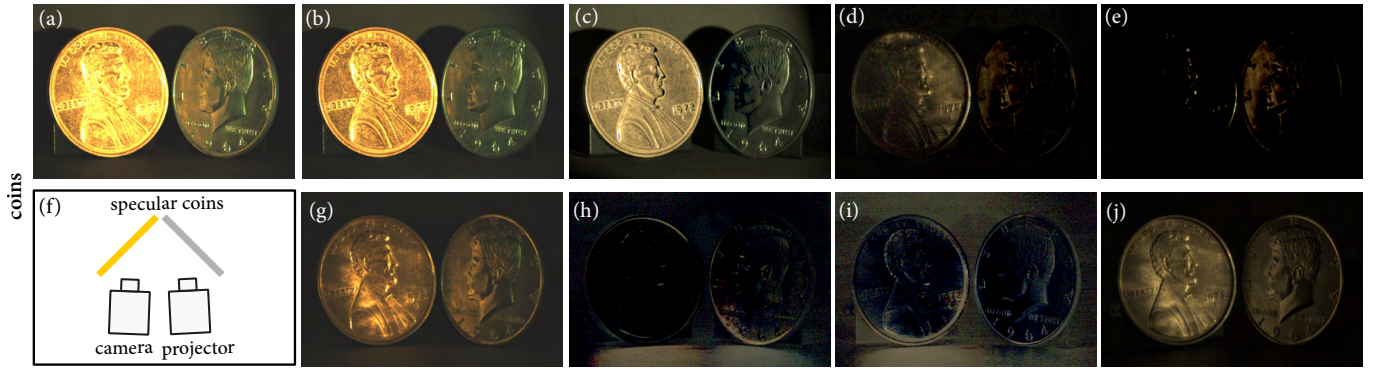


Fig. 8. We capture (a) two coins facing each other roughly at the right angle as shown in (f) the schematic diagram. Conventional (b) epipolar and (g) non-epipolar imaging decompose light transport into the approximated direct and indirect reflections. The proposed epipolar-polarimetric imaging enables us to (c) completely isolate the direct interreflection on the silver coin ($-\sum_t \mathcal{T}(s, s'_e, 3, 3, t)$), (d) extract the direct interreflection between the coins ($-\sum_t \mathcal{T}(s, s'_e, 2, 3, t)$), and (e) isolate the direct interreflection on the silver coin only ($\sum_t \mathcal{T}(s, s'_e, 3, 3, t)$). Resolving non-epipolar spatial ambiguity with polarimetric information, we isolate (h) the reflections from the coins to the floor ($-\sum_t \mathcal{T}(s, s'_n, 1, 0, t)$) or (i) only from the left coin ($-\sum_t \mathcal{T}(s, s'_n, 0, 2, t)$). (j) We also obtain the indirect interreflection between the coins ($-\sum_t \mathcal{T}(s, s'_n, 2, 3, t)$).

et al. 2018] by probing epipolar spatial and polarimetric light transport. Figure 7 shows additional reconstructions of a turtle sculpture. As seen in the epipolar-graycode capture, the strong subsurface-scattering effects make the graycode patterns invisible despite of using epipolar imaging. This degrades the reconstruction quality of structured light. Shape-from-polarization imaging combined with epipolar probing allows for recovery of fine-scale geometric details, however, this method still suffers from azimuthal ambiguity of surface normals [Baek et al. 2018]. Our epipolar polarimetric imaging accurately reconstructs both coarse and fine geometric details.

In terms of reconstruction time, graycode-based structured light requires ~ 30 seconds and shape-from-polarization requires ~ 2 seconds. Our method combines the output normals from both methods using surface-normal optimization which takes ~ 1 minute, similar to Baek et al. [Baek et al. 2018].

Spatio-polarimetric Light Transport. We show recovered spatio-polarimetric light transport in Figures 11, 12, 13, 14, 15, 16, 17. For additional visualizations of the temporal-polarimetric light transport, we refer the reader to the Supplemental Video.

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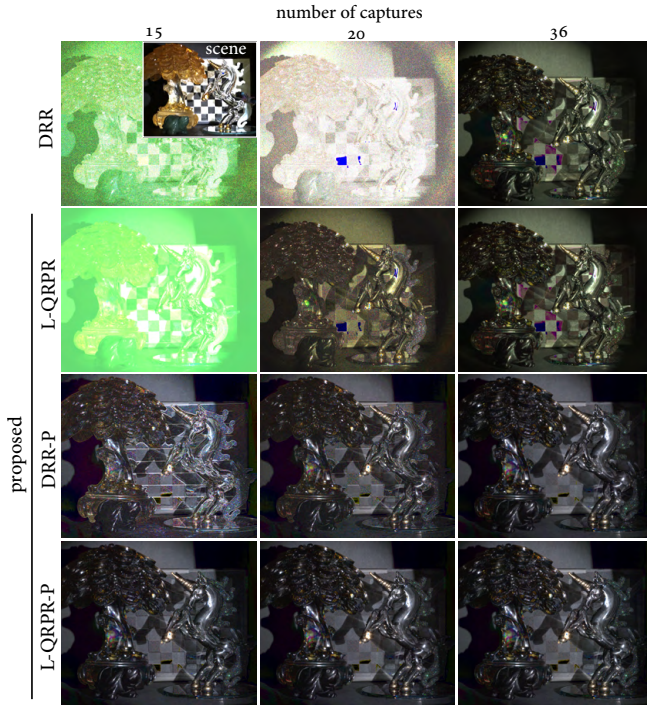


Fig. 9. Our learned ellipsometry methods can efficiently acquire complex polarimetric light transport with a small number of captures. We visualize the circular component of the reconstructed transport, $|\sum_{s',t} \mathcal{T}(s, s', 3, 3, t)|$.

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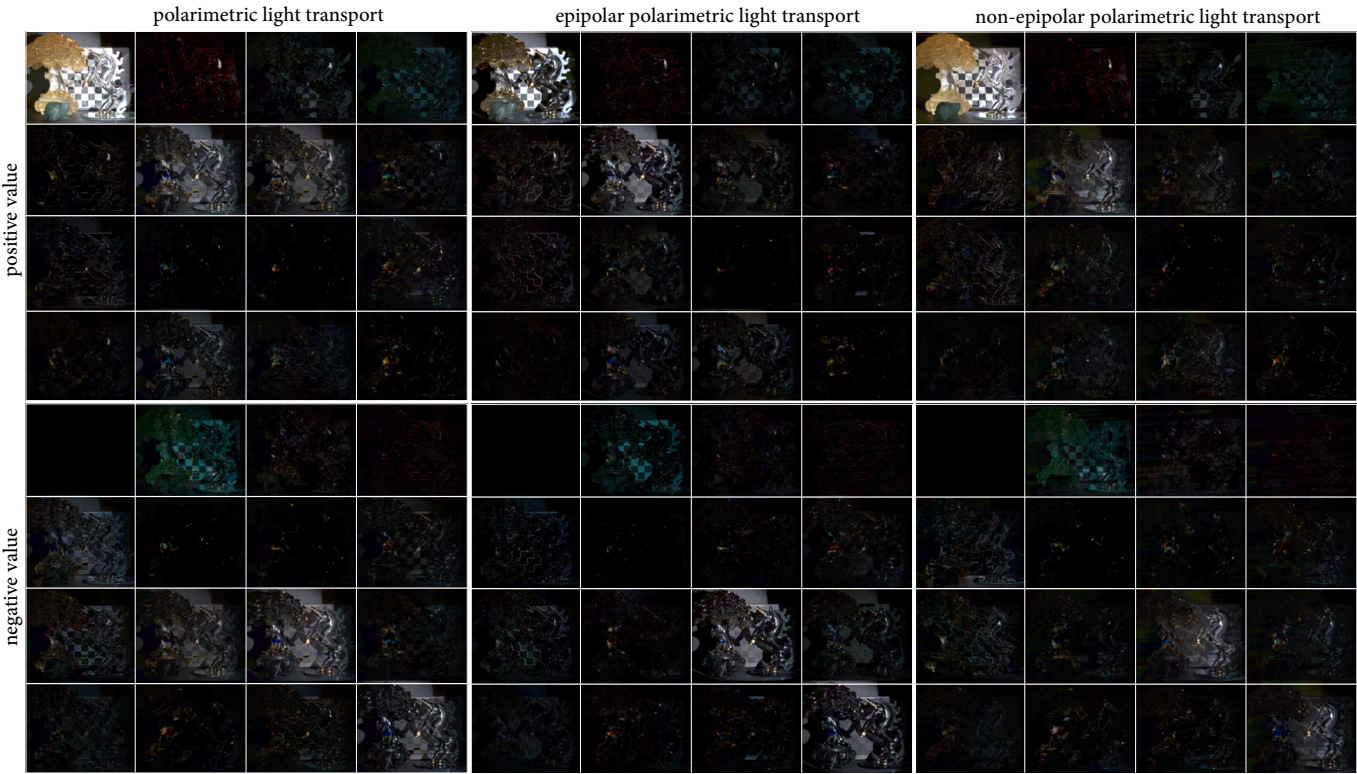


Fig. 10. We obtain the spatio-polarimetric light transport using our learned ellipsometry method with a polarizer-array sensor, L-QRPR-P, with 15 captures.

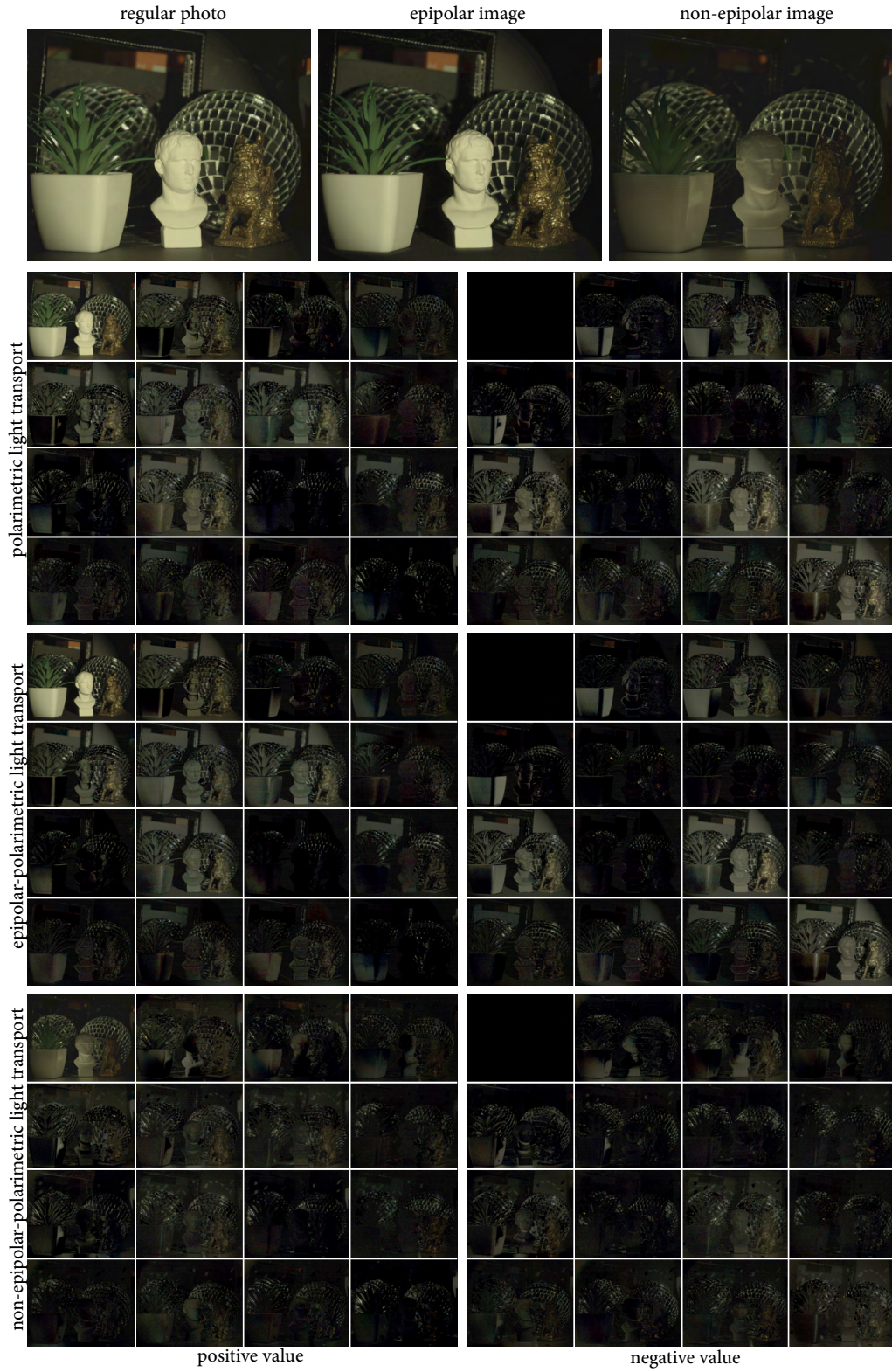


Fig. 11. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.

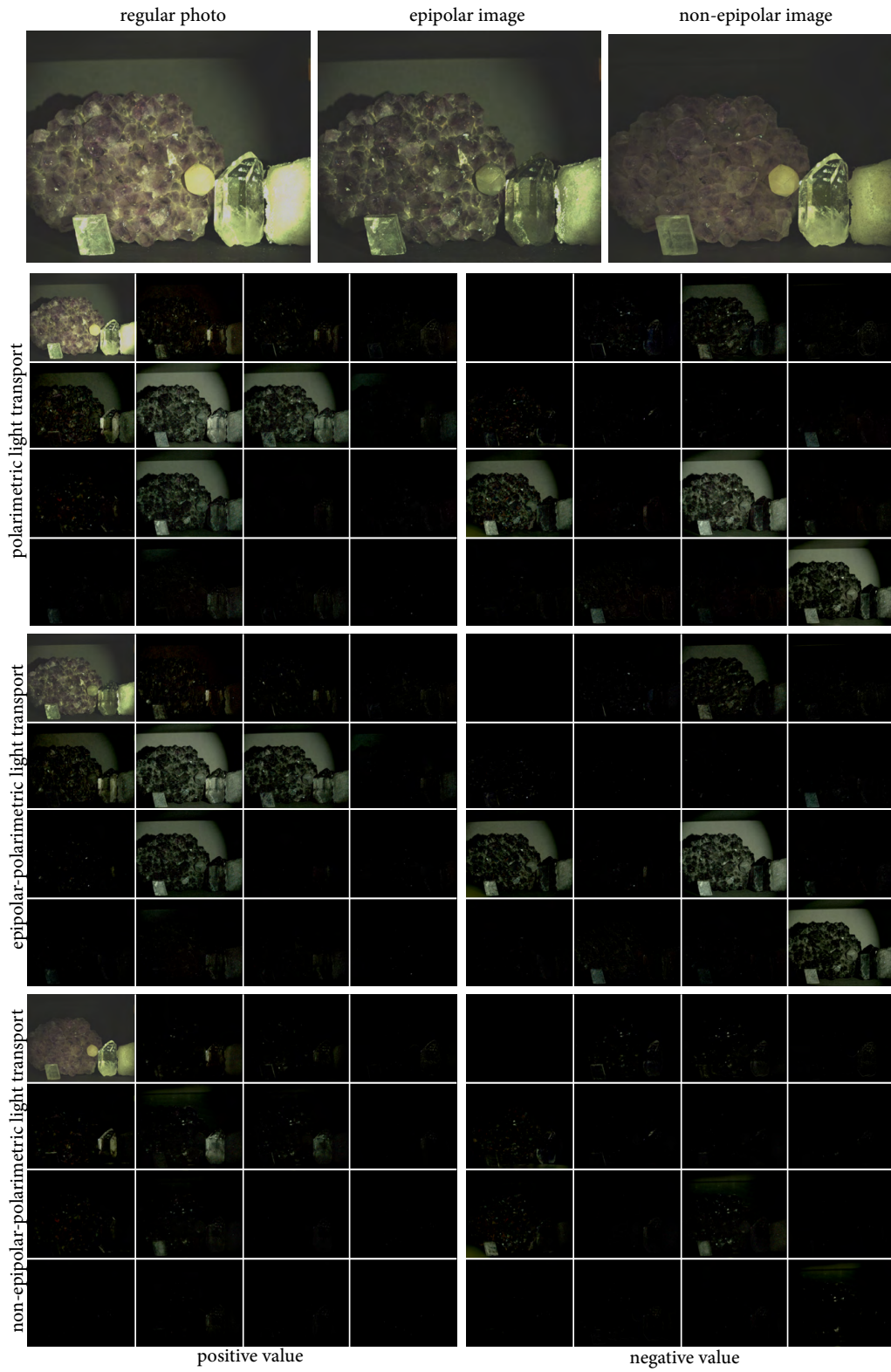


Fig. 12. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.

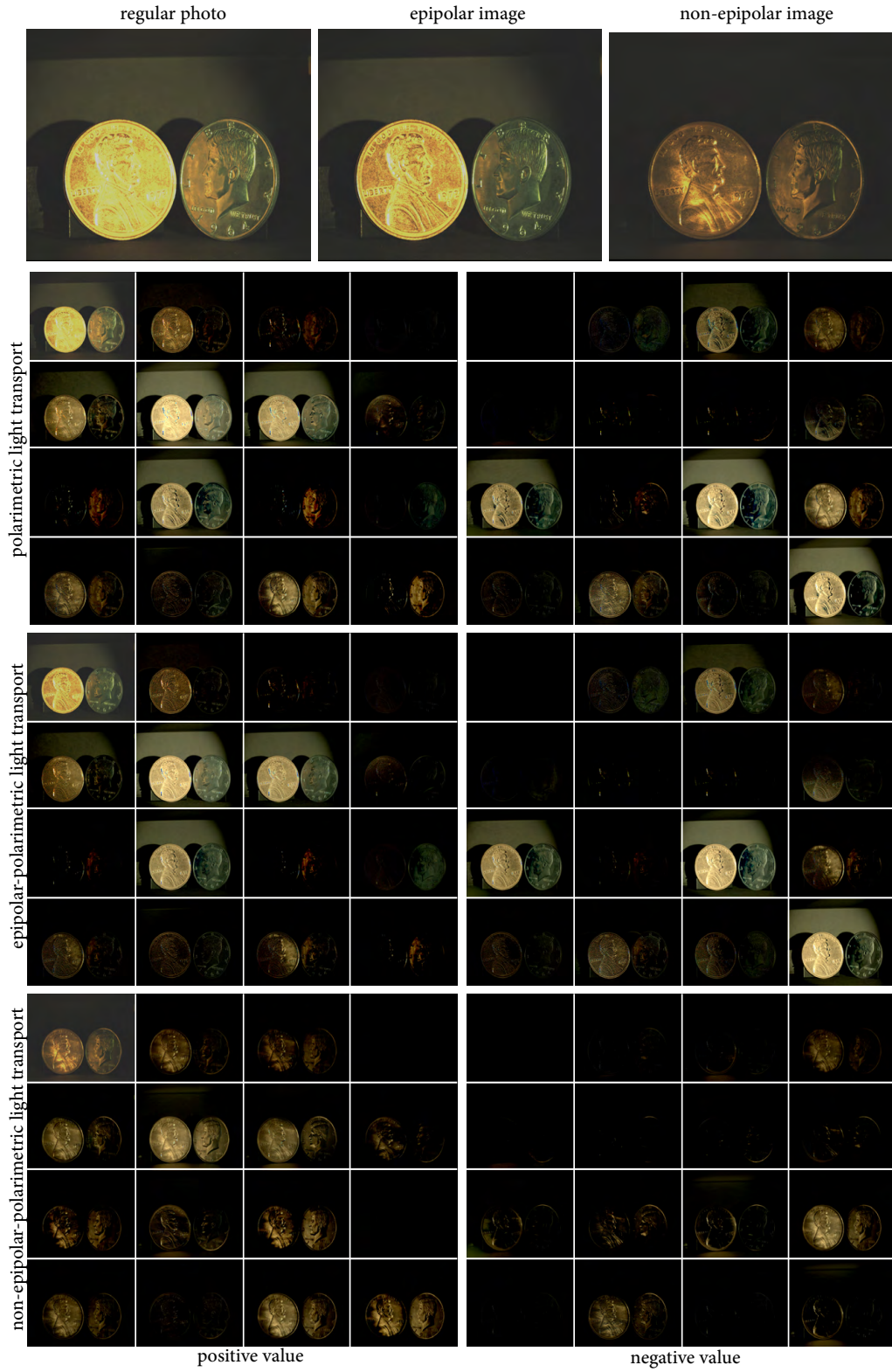


Fig. 13. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.

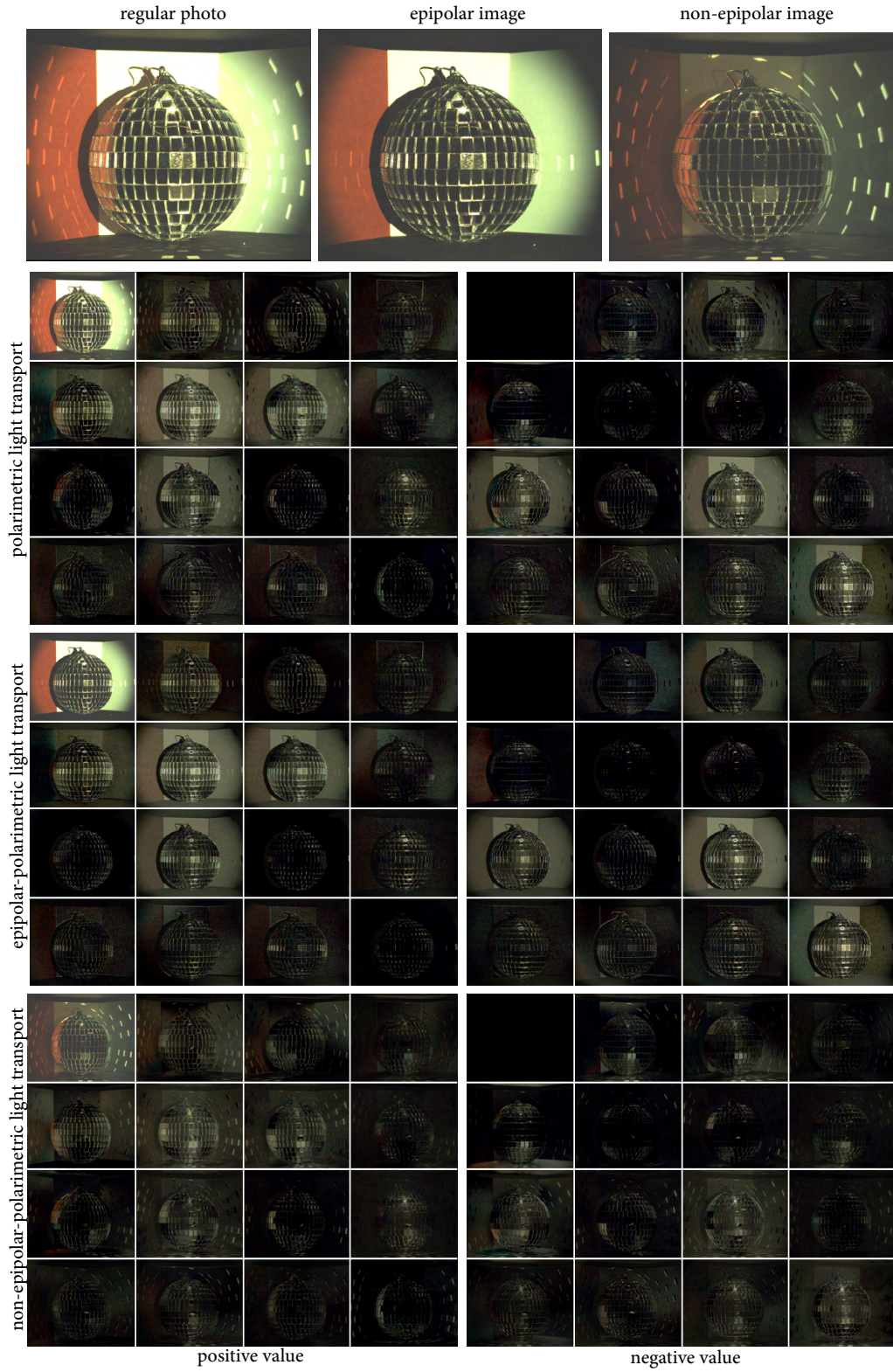


Fig. 14. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.



Fig. 15. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.

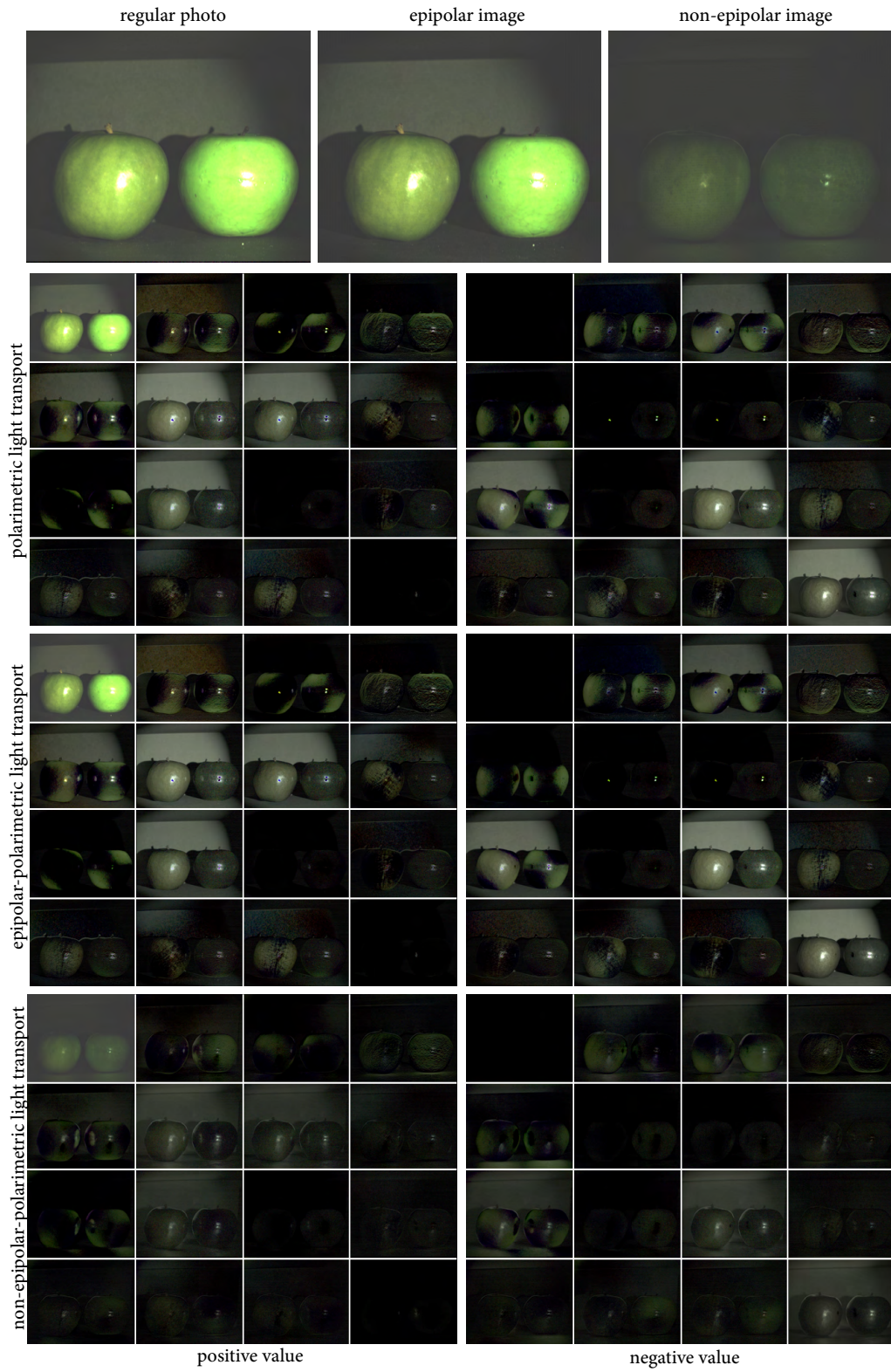


Fig. 16. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.

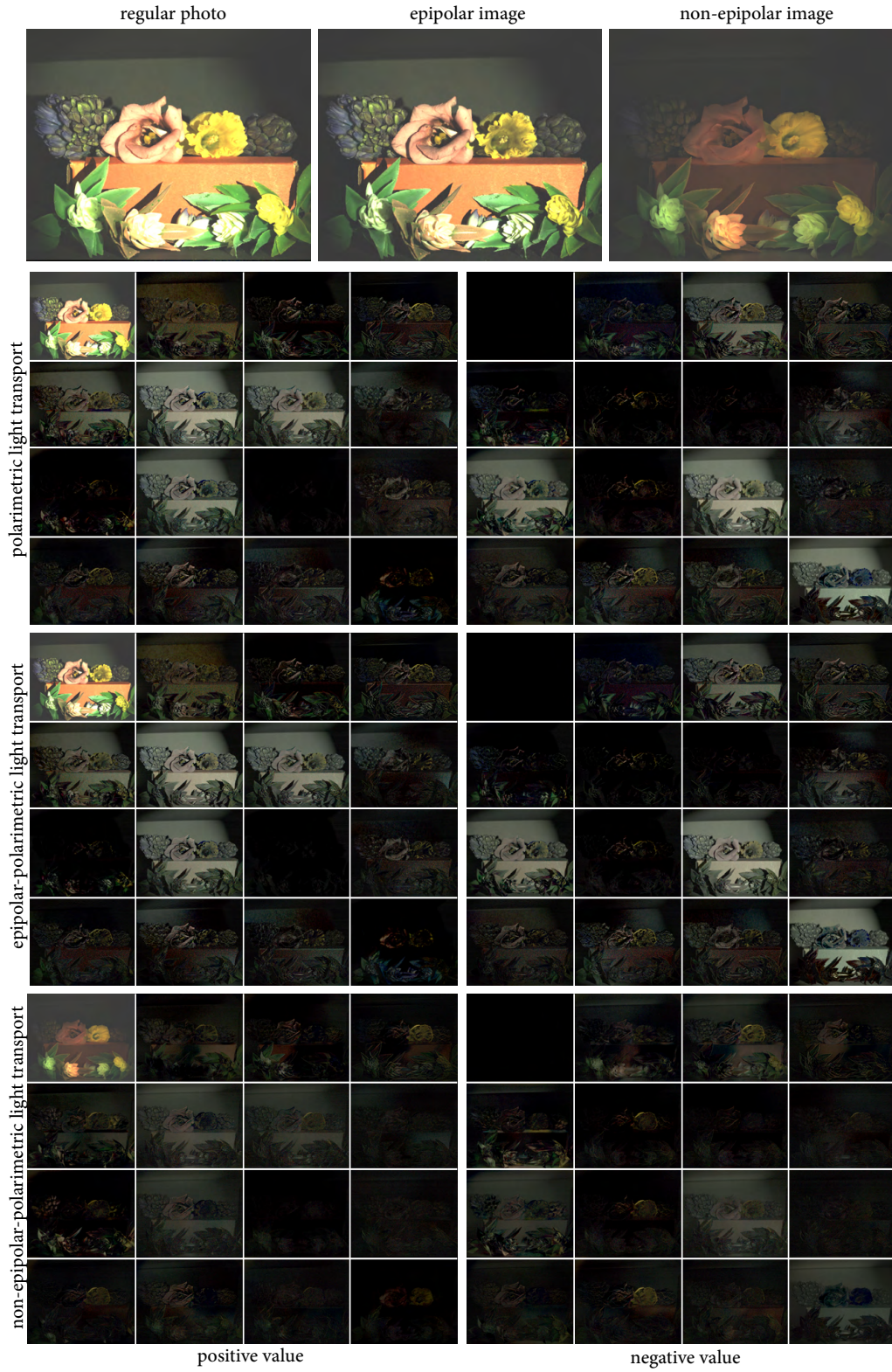


Fig. 17. Additional spatio-polarimetric decompositions. The top row shows a conventional photography and the corresponding epipolar/non-epipolar captures [O’Toole et al. 2012]. The second row shows the polarimetric reconstructions [Azzam 1978]. The bottom two rows show the epipolar- and non-epipolar-polarimetric reconstructions.